Spontaneous and stimulated emission

Consider how an atom with energy levels E_1, E_2 can interact with an electromagnetic field. The field has an energy density $u(f, T)$.

1. If in state 1, it can absorb an atom of energy

   $$E_2 - E_1 = hf$$

   making a transition from state $1 \rightarrow 2$. Let this be with probability $B_{12}u(f, T)$.

2. If in state 2, it can spontaneously emit an atom with an emission rate of A_{21}, lifetime $t_s = 1/A_{21}$. It transitions from state $2 \rightarrow 1$, *without* interacting with the field.

3. If a photon of energy hf in the field interacts with it in state 2, it can emit a photon *coherent* with the field photon. This can happen with probability $B_{21}u(f, T)$.

Now consider a mixture of atoms and radiation in equilibrium at T: The levels E_1, E_2 have populations N_1, N_2, given by Boltzmann relation:

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/k_B T} = e^{-hf/k_B T}$$

This is a *dynamic equilibrium* situation. The number of atoms per time making these transitions must balance:

$$\text{by absorption } 1 \to 2 = N_1 u(f,T) B_{12},$$

$$\text{by stimulated emission } 2 \to 1 = N_2 u(f,T) B_{21},$$

$$\text{spontaneous emission } 2 \to 1 = N_2 A_{21}.$$

Atoms going up must equal atoms going down:

$$N_1 u(f,T) B_{12} = N_2[B_{21} u(f,T) + A_{21}].$$

Put this into the Boltzmann expression

$$\frac{N_2}{N_1} = e^{-hf/k_B T}$$

and solve for the radiation density:

$$u(f,T) = \frac{A_{21}}{B_{12} e^{hf/k_B T} - B_{21}}.$$

Since this is in equilibrium, this must be the Planck law:

$$u(f,T) = \frac{8\pi h f^3}{c^3} \left(\frac{1}{e^{hf/k_B T} - 1} \right).$$

This tells us

$$B_{21} = B_{12} = B$$

so the probability of *stimulated emission* is the same as that for absorption, and

$$\frac{A_{21}}{B} = \frac{8\pi h f^3}{c^3}.$$

Spontaneous emission dominates for high f.

Under *equilibrim thermal* conditions*

$$\frac{N_2}{N_1} = e^{-hf/k_B T} < 1 \text{ since } T > 0,$$

so absorption by atoms in lower states dominates.

*A *negative* T can be defined for strongly interacting systems with an upper energy limit.

Population inversion

If atomic energy level populations can be arranged so that there are more atoms in upper states than lower states, amplification of photons can occur.

Requirements for laser action

- An energy source that can produce population inversions.

- A lasing medium with at least *three* E levels: a ground state, a long-lifetime intermediate, and a high-E pump state. Transitions from the pump state produce a population inversion in the long-lifetime state, which then undergoes stimulated emission.

  A two level system only gets $N_2 \leq N_1$ even for extreme pumping because $B_{21} = B_{12}$ for a *given* transition.

- A method of containing the initially emitted photons so they can stimulate further emission. This is usually done by making an optical resonator with mirrors.