Nuclear Structure continued

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Nuclear size

"I would have been no more surprised if someone fired a 15-inch artillery shell at tissue paper and had it bounce back." —Rutherford

J. J. Thomson had proposed that electrons would be slightly deviated on passing through matter by a positive charge distributed around inside an atom, with electrons embedded in it as point charges—the "plum pudding" model.

Rutherford found that high speed electrons and α particles (He nuclei) were sometimes deflected by as much as 180 degrees (see quote above)—evidence for a very small "hard-core" nucleus. (1910-1911.)

By equating the incoming KE to the electrical PE for such a scattering event, Rutherford estimated the nuclear size at a radius of about 10^{-14} m.

A common nuclear distance unit is the femtometer, or **fermi:**

$$1 \text{ fm} = 10^{-15} \text{ m}.$$

Further work has shown that nuclear radii are approximately spherical of radius:

$$r = r_0 A^{1/3}$$

where A is the mass number and $r_0 = 1.2$ fm.

The volume $\propto A$, so *all nuclei have approximately the same density.*

The neutron

By the 1920s it was known that a nucleus had Z charged protons and a mass of about A protons, with $A \approx 2Z$. Rutherford proposed $A-Z$ neutral $p-e$ combinations as "neutrons".

Free neutral particles of mass about $1m_p$ were discovered by Chadwick in 1932—neutrons. The idea that they are $p-e$ combinations has been dropped because of spin and energy considerations. In β decay a proton and electron are *created* by the energy of the neutron.

A free neutron decays to a proton and electron with a half-life of about 10 minutes.

Protons and neutrons together are often called **nucleons.**

Nuclear stability

The **nuclear force** is a very short range (~ 2 fm) attractive force that acts on nucleons. Electrostatic forces produce repulsion between protons, while the nuclear force produces a competing attraction between them. Remember that electrostatic forces are long-range.

There are ≈ 260 stable nuclei. Light nuclei are most stable when $N \approx Z$.

As Z increases, more neutrons are needed to hold the nucleus together, as they produce only attractive forces. For heavy nuclei $N > Z$.

For nuclei of $Z > 83$, there are no stable isotopes.

Magic numbers

Most stable nuclei have A *even.*

Certain values are so-called **magic numbers:**

$$Z \text{ or } N = 2, 8, 20, 28, 50, 82, 126$$

which correspond to high stability in nuclei.

Nice pdf table of nuclides at:

`http://ie.lbl.gov/toi/pdf/chart.pdf`

Binding energy

The total mass of a nucleus is always *less* than the mass of its nucleons. Mass is a measure of energy ($E = mc^2$), so **the total energy of the bound system (nucleus) is less than the combined energy of the separated nucleons.**

This difference is the **binding energy**:

$$E_b(\text{MeV}) = [ZM(\text{H}) + Nm_n - M_A] \times 931.494\text{MeV/u}$$

$M(\text{H})$ is the atomic mass of hydrogen, M_A is the atomic mass * of ${}^{A}_{Z}\text{X}$, m_n is neutron mass, all masses in u.

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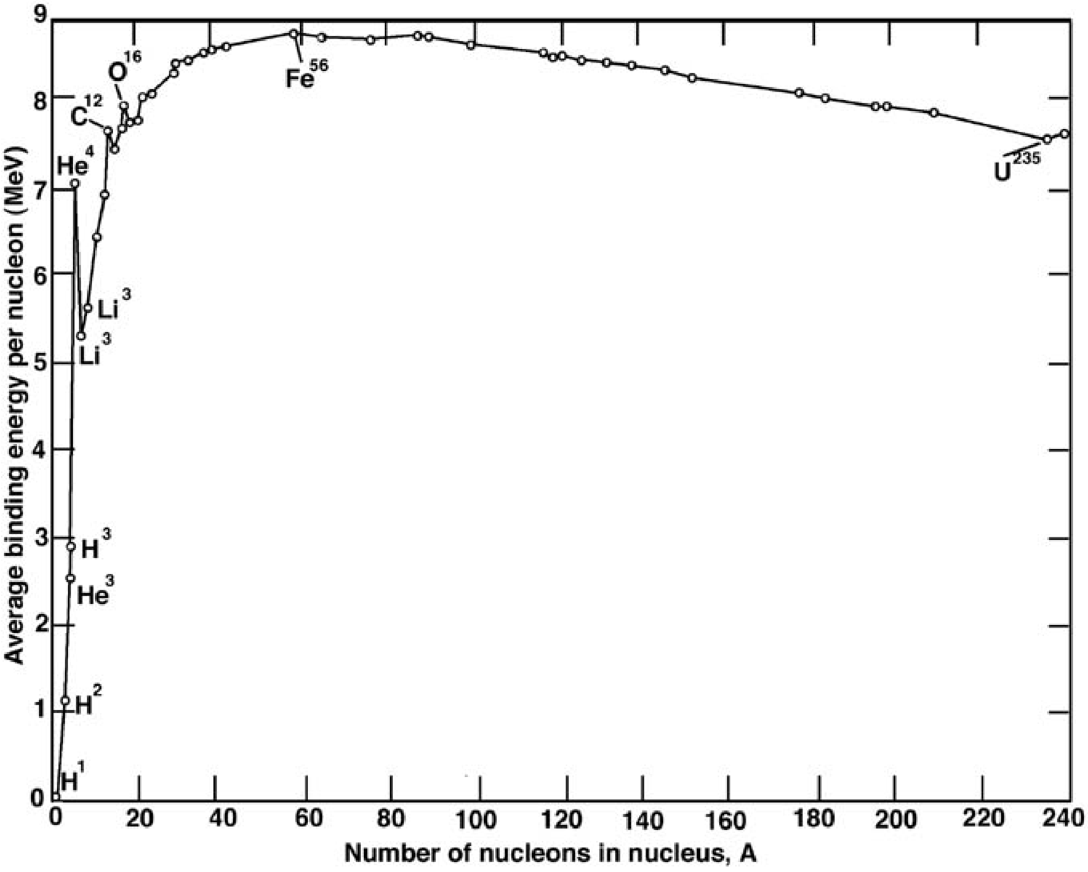
*Using atomic masses cancels out electron masses.

Binding energy per nucleon

If E_b/A is plotted as function of A, we see that the nuclei around $A = 60$ (Fe, Co) are the most tightly bound. In nuclear reactions, the products will "climb" toward maximum E_b/A.

- For nuclei of $A \approx 200$, energy will be released on splitting into smaller fragments (fission).

- For nuclei of $A \leq 20$, energy will be released by combining nuclei (fusion).



Average binding energy per nucleon (MeV)
Number of nucleons in nucleus, A
0
1
2
3
4
5
6
7
8
9
0
20
40
60
80
100
120
140
160
180
200
220
240
H1
H2
He3
H3
Li3
Li3
He4
C12
O16
Fe56
U235