Mathematics of Radioactivity; Alpha Decay

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Most common forms of nuclear radiation

- Alpha (α) particles—He nuclei $A = 4, Z = 2, N = 2$ (doubly magic).
- Beta (β) particles—electrons (e^-) or positrons (e^+).
- Gamma (γ) photons—very high-energy electromagnetic radiation.

These can be discriminated by their charges and energies using magnetic fields. One can also make pretty good determinations by their ability to penetrate bulk matter (Gammas most, Alphas least.)

Decay rates

For any given radioactive nuclide, each nucleus has a fixed probability of decaying in a given time.

The number decaying is proportional to the number in the sample:

$$\frac{dN}{dt} = -\lambda N.$$

where λ, the **decay constant** is probability/time that a nucleus will decay. N is *decreasing* in time.

We can rearrange the differentials:

$$\frac{dN}{N} = -\lambda \, dt$$

and integrate

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t \, dt$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

or

$$\boxed{N = N_0 e^{-\lambda t}}$$

where N_0 is the number of radioactive nuclei at $t = 0$. **The number of radioactive nuclei (of a given nuclide) in a sample declines exponentially with time.**

Decay rate

We can differentiate with respect to time to get the decay rate:

$$\boxed{R = \left|\frac{dN}{dt}\right| = N_0\lambda e^{-\lambda t} = R_0 e^{-\lambda t}}$$

where $R_0 = N_0\lambda$ is the initial decay rate. Note that $R = \lambda N$.

The decay rate R of a sample is also known as its **activity.**

Two common units of activity:

curie 1 Ci $\equiv 3.7 \times 10^{10}$ decays/s (based on 1 g of Ra)

becquerel (SI) $1\ \mathrm{Bq} \equiv 1$ decay/s.

A concentrated 1 Ci source can be dangerous!

Half-life, $T_{1/2}$

The time it takes half of a given number of radioactive nuclei to decay.

Set $N = N_0/2$ and $t = T_{1/2}$ in the expression for $N(t)$, we get

$$\boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

The **mean lifetime** for any one nucleus is just $\tau = 1/\lambda$, so $T_{1/2} = \tau \ln 2$.

Alpha decay

A nucleus emits an Alpha (He nucleus):

$$^{A}_{Z}\text{X} \rightarrow\ ^{A-4}_{Z-2}\text{Y} +\ ^{4}_{2}\text{He}$$

X is the **parent nucleus** and Y is the **daughter nucleus**. For example:

$$^{238}_{92}\text{U} \rightarrow\ ^{234}_{90}\text{Th} +\ ^{4}_{2}\text{He}$$
$$^{226}_{88}\text{Ra} \rightarrow\ ^{222}_{86}\text{Rn} +\ ^{4}_{2}\text{He}$$

The half-lives for these are 4.47×10^9 years and 1.60×10^3 years.

The **disintegration energy** Q is:

$$Q = (M_X - M_Y - M_\alpha)c^2$$
$$Q = (M_X - M_Y - M_\alpha) \times 931.494\ \text{MeV/u}$$

This appears as KE of Y and α produced in the decay. Must have $Q \geq 0$ for spontaneous decay.

For alpha emitters, $Q < 0$ for the process of emitting a single proton or neutron, but $Q > 0$ for emitting a He nucleus. Remember He has a large E_b!

Alpha decay can be modeled as a **tunelling process.** See figs. 13.17,18, and example 13.9.

We'll continue with **beta decay** in the next set of notes.