Question #47. Is there an "iso-" name for a $\Delta U=0$ process?

Thermodynamics uses the following terms to indicate certain conditions for processes:

isobaric	$\Delta P=0$
isochoric	$\Delta V=0$
isenthalpic	$\Delta H=0$
isentropic	$\Delta S=0$

Is there an equivalent "iso-" word for a process in which $\Delta U=0$? (and, for that matter, $\Delta A=0$ and $\Delta G=0$?)

David W. Ball
Department of Chemistry
Cleveland State University
Cleveland, Ohio 44115

Answer to Question #19 ["Noether's theorem and discrete symmetries," Dwight E. Neuenschwander, Am. J. Phys. 63(6), 489 (1995)]

The answer to this question is negative: As far as today is known, there is not a result of the generality and scope of Noether's Theorem for discrete symmetries. The number and importance of consequences holding from invariance of a system under a discrete group of transformations are, in fact, very limited when compared with its continuous counterpart.

One of the few exceptions to the previous statement is, without doubt, Bloch's theorem,[1] which deduces the functional dependence of the eigenstates and eigenvalues of a Hamiltonian in a periodic potential, like that of a solid. The invariance of Schrödinger's equation under an infinite but discrete group of translations (those corresponding to the underlying Bravais lattice) leads to the fundamental concept of *band structure*, which has been of great importance, both theoretical and technological. However, the eigenstates of the Hamiltonian are not, in general, simultaneous eigenstates of the momentum operator unless the size of the periodicity tends toward zero: In this case we have a constant potential, namely, a free particle. This is analogous to momentum conservation in classical mechanics:[2] It can be seen as the result of a perfect translational symmetry, that is to say, the invariance under a (continuous) group of transformations, that of translations in $\mathfrak{R}^3$.

I think that this shows how the continuity of the group is essential for the existence of an associated conservation law to hold. Otherwise, the invariance under a discrete group of transformations does not necessarily imply such a law, as the previous example illustrates.

Benito Hernández-Bermejo
Departamento de Física Fundamental
Universidad Nacional de Educación a Distancia
Apartado 60.141, 28080 Madrid (Spain)

[1]See, for example, N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders, Philadelphia, 1976), pp. 133–141.
[2]L. D. Landau and E. M. Lifshitz, *Mechanics* (Pergamon, Oxford, 1976), 3rd ed., pp. 15 and 16.

Answer to Question #19 ["Noether's theorem and discrete symmetries," Dwight E. Neuenschwander, Am. J. Phys. 63(6), 489 (1995)]

The question is whether there is an analog to Noether's theorem, relating symmetries to conservation laws, for the case of discrete symmetries. There is, in fact, a most elegant generalization, in the context of quantum mechanics, which is applicable to all symmetries, discrete and continuous, that are associated with invariance under unitary transformations. The only exceptions, then, are symmetries that include time reversal.

Every Hermitian linear operator in quantum theory (leaving aside mathematical subtleties) plays a dual role: It represents a physical observable, and it serves as the generator of a set of transformations. When two operators commute, the physical meaning takes different forms depending on the role assigned to each. In particular, if A and B are two observables, and $\hat{A}$ and $\hat{B}$ are the corresponding operators, then $\hat{A}$ and $\hat{B}$ commute if and only if the observable A is invariant under the transformations generated by $\hat{B}$. Reversing the roles of A and B, and combining, you get the following very symmetrical rule:

The observable A is invariant under the transformations generated by $\hat{B}$ if and only if the observable B is invariant under the transformations generated by $\hat{A}$.

You get Noether's theorem by letting $\hat{B}$, say, be the Hamiltonian operator $\hat{H}$. The transformations generated by $\hat{H}$ are simply time displacements, and invariance under time displacements corresponds to a conservation law. The invariance of $\hat{H}$ under transformations generated by $\hat{A}$, on the other hand, corresponds to a symmetry of nature, i.e., an invariance of the equations of motion generated by the Hamiltonian. The specific form of Noether's theorem is thus:

The observable A is conserved if and only if the equations of motion are invariant under the transformations generated by the corresponding operator $\hat{A}$.

In the case of a continuous symmetry, the Hermitian operator $\hat{A}$ is related (by an exponential function) to the unitary transformations in question. In the case of a discrete symmetry the operator $\hat{A}$ has to be unitary, and the observable A is a unimodular, rather than real, variable. The most common discrete symmetries, though, are simple inversions like parity and charge conjugation, and $\hat{A}$ itself is then both Hermitian and unitary.

In the context of classical Hamiltonian mechanics the identical argument can be carried through, for continuous symmetries only, by replacing commutators by Poisson brackets. This is because infinitesimal canonical transformations can be generated by physical observables, while discrete finite transformations cannot.

Robert Mills
Department of Physics
174 W. 18th Avenue, Ohio State University
Columbus, Ohio 43210

Answer to Question #21 ["Snell's law in quantum mechanics," Steve Blau and Brad Halfpap, Am. J. Phys. 63(7), 583 (1995)]

The question of how to interpret Snell's law and the index of refraction from the point of view of photons and quantum mechanics can usefully be recast as a question of how to interpret these concepts from a microscopic point of view, whether quantum-mechanical or (semi-)classical. Feynman has an excellent microscopic analysis of the index of refraction in his Chapter 31 on "The Origin of the Refractive Index."[1] He points out that, "*so far as problems involving light are concerned,* the electrons (in atoms) behave as though they were held by springs" (p. 31-4). One can glimpse how this is possible by approximating the electron cloud in hydrogen with a uniform-density sphere of radius R. If the proton is displaced a distance s from the center of this sphere, there is a restoring force on the proton due to the electric field contributed by that portion of the sphere that is inside the radius s:

$$F = q_{\text{proton}} E_{\text{electron sphere}} = e \frac{1}{4\pi\epsilon_0} \left[\left(\frac{e}{\frac{4\pi}{3} R^3} \right) \frac{4\pi}{3} s^3 \right] \frac{1}{s^2} = \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{R^3} \right) s. \tag{1}$$

This is also the force exerted on the electron cloud by the proton, and it is proportional to s, just like a spring force. Of course quantum mechanics is required to predict the actual electron charge distribution, but many of the electric consequences of that charge distribution can be analyzed classically.

In a microscopic but otherwise classical analysis, the electric field in electromagnetic radiation accelerates electrons held by springs in the atoms of a piece of glass, and these accelerated electrons re-radiate in all directions. The observed light is the superposition of the electric (and magnetic) fields of the incoming light and the re-radiation. Full quantitative analysis from a microscopic point of view requires a kind of self-consistent calculation, because the re-radiation from accelerated electrons contributes to the net electric field driving the electrons. Feynman deals with the low-density limit in which re-radiation of re-radiation is negligible, but this is adequate to understand the essential aspects of the phenomena. In the backward direction we normally call the re-radiation "reflection," but this labeling obscures the fact that this is *new* light radiated by *all* the atoms in the glass, not old light that has magically "bounced off" the front surface due to some unknown mechanism. The microscopic analysis of "reflection" is exactly the same as the analysis of x-ray diffraction, but because the interatomic spacing is small compared to the wavelength of visible light, the "reflected" light has just one, zeroth-order interference maximum in the "reflection" direction ($n=0$ in the Bragg "reflection" condition).

In the forward direction we speak of "refraction," and we say that "the speed of light is slower in the glass," but, in fact, *the speed of light does not change in the material.* Rather, Feynman shows how the superposition of the incoming light, traveling at speed c, and the light re-radiated by the atomic electrons, traveling at speed c, shifts the phase of the radiation in the air downstream of the glass in the same way that would occur if the light were to go slower than c in the glass, with a shorter wavelength and an index of refraction greater than 1 for frequencies below the natural frequency of the oscillators (otherwise the phase shift corresponds to a speed greater than c in the material, with index of refraction less than 1). At a fundamental level this phase velocity, greater or less than c, is of no particular physical significance, because it only applies to the overly simplified case of single-frequency sinusoidal radiation permeating all space, and such radiation cannot carry a meaningful signal.

To see what happens to a meaningful signal, consider incoming radiation in the form of a sine wave that starts suddenly. Detecting the leading edge of this sine wave provides real information. Suppose the electric field in the first half cycle is in the $+y$ direction. An electron is initially driven downward by the incoming electric field, and the accelerated electron radiates in (nearly) all directions. In the forward direction the re-radiated field at far distances is proportional to $-qa_y = +ea_y$ (with a_y negative), so that the contribution to the net downstream field is in the $-y$ direction. Therefore, at a downstream observation point, the *net* field during the beginning of the cycle is reduced slightly from what it would be in the absence of the charge on a spring, and the rise time is slower.

The details of the net wave shape can be studied by numerical integration of the motion of the electron on the spring under the influence of the sudden-onset incoming sine wave, to obtain the acceleration of the electron as a function of time to use in evaluating the charge's $-qa_y$ contribution to the net field. One finds by numerical computation that the effect of a single oscillator is to make the initial maximum of the *net* field at the observation point occur slightly late, as though the speed of light were less than c.

However, the apparent "slower speed" is the result of the superposition of two radiative electric fields, the incoming radiation and the re-radiation, *both of which travel at the normal speed of light c.* If taken too seriously, it is a violation of the superposition principle to say that the speed of light is affected by the presence of matter. The incoming radiation was produced by some accelerated charges, and the field that those charges produced is unaffected by the presence of other charges anywhere in the universe, and this field propagates at speed c. In particular, incoming radiation passes through glass *unchanged,* but downstream we observe the superposition of this unchanged radiation with re-radiation from the accelerated electrons in the glass. The leading edge of radiation may travel at a speed smaller than c, but only through the superposition of the contributions of accelerated charges that make radiative fields that propagate at speed c.

Another relevant computation is to re-do numerically Feynman's analytical calculation for a very thin glass plate, but with a sudden-onset sine wave instead of a continuous sine wave. For a driving frequency below the natural frequency of the oscillators, one finds a slight delay in the first maximum, and one also finds in agreement with Feynman's calculation a phase delay after the steady state is attained. For a driving frequency above the natural frequency of the oscillators, there is hardly any delay in the first maximum, and there is a phase *advance* after the steady state is attained, as discussed by Feynman in Section 31-4 on "Dispersion."

It is enormously convenient to describe refraction by saying "the speed of visible light is smaller in glass." It would be extremely difficult from the microscopic viewpoint to calculate the index of refraction for a dense material such as glass, due to having to take into account re-radiation of re-radiation in a self-consistent way, using the correct form of the retarded fields of nearby accelerated electrons (Feyn-

man's calculation not only neglects re-radiation of re-radiation but requires only the far-field approximation). The index of refraction lumps all of this complexity into one convenient number, and one convenient metaphor. The mathematical complexity of the microscopic analysis is prohibitive for most quantitative work, but it complements the macroscopic picture by providing a deep sense of mechanism and by permitting a unified microscopic analysis of reflection, refraction, x-ray diffraction, and even thin-film interference. This is analogous to the insight that kinetic theory adds to thermodynamics, or that circuit analysis in terms of surface charge[2] adds to the Kirchhoff loop and node rules.

The original question asked about Snell's law from the point of view of photons. The main issue is not really photons, but microscopic versus macroscopic analyses. The passage to quantum mechanics introduces still more mathematical complexity but does not change the main point. The reflected and refracted light consists of the (quantum) interference of incoming photons with photons re-emitted by atoms in the glass. The fundamental speed of light is unaffected.

Bruce A. Sherwood
Center for Innovation in Learning and Department of Physics
Carnegie Mellon University
Pittsburgh, Pennsylvania 15213

[1]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1963), Chap. 31. See also, for instance, Mary B. James and David J. Griffiths, "Why the speed of light is reduced in a transparent medium," Am. J. Phys. **60**(4), 309–313 (1992).
[2]R. W. Chabay and B. A. Sherwood, *Electric and Magnetic Interactions* (Wiley, New York, 1995), Chap. 6. Also see Chap. 14 on the classical interaction of light and matter.

Answer to Question #21 ["Snell's law in quantum mechanics," Steve Blau and Brad Halfpap, Am. J. Phys. 63 (7), 583 (1995)]

Concerning the interpretation of reflection and refraction in terms of photons,[1] it should first be noted that the notion of "billiard ball" photons very often leads to confusion,[2] especially if it is used to describe the propagation and interference of light independently of and prior to its detection. Interference effects, for instance, really involve the interference of quantum-mechanical probability amplitudes (whose squared moduli are the probabilities) rather than photons *per se*.

In the quantum-mechanical description of a plane wave incident on a dielectric medium, each photon has a probability amplitude to be scattered by any one atom. The complete probability amplitude for a photon to be found at any point inside or outside the medium is the amplitude for it to get there without any scattering, plus the sum over all the possible paths by which it can get there via single- and multiple-atom scattering. The result of this superposition of all possible probability amplitudes is an amplitude that is nonzero both inside and outside the medium. The part outside the medium is nonvanishing only in directions given by the law of reflection, while the part inside is nonvanishing only in directions given by Snell's law; the latter part propagates at the phase velocity of light in the medium. The probabilities for reflection and refraction are exactly the reflection and transmission factors given by the Fresnel formulas for the field intensity. This result has been worked out explicitly for the case of normal incidence of a single photon.[3]

Actually the situation can be described very similarly in classical electromagnetic theory. According to the superposition principle, the field at any point is the sum of the incident field plus the fields produced by scattering from the dipoles of the medium, each field propagating at the vacuum velocity of light. The results of this superposition are the usual reflected and refracted waves whose directions are given by the laws of reflection and Snell. This is the essence of the Ewald–Oseen extinction theorem of classical optics.[4]

In other words, the probability amplitude for finding a photon at any point in this example follows exactly the Maxwell equations; for most purposes we can, in fact, regard the vector potential as in effect a wave function for a photon.[5] The important point is that the photon wave function here represents the probability of finding a single photon. When the number of photons is large, we can regard this wave function as a directly measurable, classical wave, and explain the classical Snell's law as a consequence of interfering probability amplitudes. Indeed "the first observations were on situations with many photons in the same state, and so we were able to discover the correct equation for a single photon by observing directly with our hands on a microscopic level the nature of wave function."[5]

P. W. Milonni
Theoretical Division (T-4)
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

[1]S. Blau and B. Halfpap, Am. J. Phys. **63**, 583 (1995), Question #21.
[2]See, for instance, W. E. Lamb, Jr., "Anti-Photon," Appl. Phys. B **60**, 77 (1995); R. J. Glauber, "Dirac's Famous Dictum on Interference: One Photon or Two?," Am. J. Phys. **63**, 12 (1995); P. W. Milonni, "Wave-Particle Duality of Light: A Current Perspective," in *Wave-Particle Dualism, Papers in Honor of the 90th Birthday of Louis de Broglie,* edited by S. Diner, D. Fargue, G. Lochak, and F. Selleri (Reidel, Dordrecht, 1983).
[3]R. J. Cook and P. W. Milonni, "Quantum Theory of an Atom Near Partially Reflecting Walls," Phys. Rev. A **35**, 5081 (1987).
[4]M. Born and E. Wolf, *Principles of Optics* (Pergamon, London, 1980), 6th ed.
[5]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1965), Vol. III, p. 21-6.

Answer to Question #30 ["How are positrons moderated?," Thomas D. Rossing, Am. J. Phys. 63(12), 1065 (1995)]

The current technology[1] for producing low-energy positron beams uses a high-purity, thin crystalline metallic foil (typically on the order of microns thick) which is illuminated on one side by a radioactive positron source, typically ^{22}Na. The emission of positrons from the far side of the foil is related to several fortunate characteristics regarding the behavior of positrons in metals. First, positrons are able to thermalize in a metal in about 10 ps, which is an order of magnitude smaller than the mean lifetime of positrons in the metal. Second, on the order of 1% of the positrons incident on the source side of the metal foil will stop within a distance of the far side comparable to the diffusion length of a thermal positron during its mean lifetime. Thus a significant number of positrons can actually make it to the surface on the other side of the foil. It is here that the energetics of the metal surface play an important role. Positrons at the surface