Pressure, Radiation

D. Craig

2011-01-28

Radiation pressure

Electromagnetic waves (or photons) carry momentum.

Since the irradiance is energy/area/time, pressure is the energy density of a wave. We can express this with the average magnitude of the Poynting vector S.

$$\langle \mathcal{P}(t) \rangle_T = \frac{\langle \mathcal{S}(t) \rangle_T}{c} = \frac{I}{c}$$

in N/m^2. This is for a perfect absorber. For a perfect reflector it is $2I/c$ (remember elastic collisions?)

Each photon carries an energy $\mathcal{E} = h\nu$. The photon will carry momentum

$$p = \frac{\mathcal{E}}{c} = \frac{h}{\lambda}.$$

In vector form

$$\vec{\mathbf{p}} = \hbar\vec{\mathbf{k}}$$

$\vec{\mathbf{k}}$ is the propagation vector and $\hbar \equiv h/2\pi$.

Effects of radiation pressure. Average flux of energy from the Sun at Earth's orbit is 1400 W/m^2. This gives a pressure of about 9 N/km^2.

This is sufficient pressure to affect the orbit of long-range spacecraft, and must be accounted for in mission planning. It is also possible to levitate very small objects using radiation in the laboratory (see fig. on p. 57). A sail of about a square km or more could be used to accelerate a few tons across the inner solar system.

In stellar interiors, the radiation pressure is a major part of the structure of the star (such as our Sun).

Radiation—if a charge moves nonuniformly, it radiates.

(See figs. 3.27–28.)

The radiation fields (large r) due to acceleration go as

$$\vec{\mathbf{E}} \sim \vec{\mathbf{r}} \times (\vec{\mathbf{r}} \times \vec{\mathbf{a}}) \text{ and}$$
$$\vec{\mathbf{B}} \sim \vec{\mathbf{a}} \times \vec{\mathbf{r}},$$

where $\vec{\mathbf{r}}$ points from the charge and $\vec{\mathbf{a}}$ is the acceleration vector.

This means that **energy is most strongly radiated perpendicular to the acceleration causing it.**

Synchrotron radiation

Any curved path means an acceleration.

A free charge on a curved path will radiate.

This is important astronomically due to charges in stellar and interstellar magnetic fields, and a charged particle accelerator driven as a *synchrotron source* can produce intense X-rays. See fig. 3.30.

Electric dipole radiation

(See fig. 3.32.)

A dipole of charge with static dipole moment $\widehat{p_0} = qd$ oscillating at angular frequncy ω produces a radially outward irradiance

$$I(\theta) = \frac{\widehat{p_0}\omega^4}{32\pi^2 c^3 \epsilon_0} \frac{\sin^2\theta}{r^2},$$

for $r \gg d$, the *radiation zone*.

Note that

$$I \sim \omega^4 \sim \nu^4 \sim \frac{1}{\lambda^4}.$$

Scattering of radiation by the dipole moment of air molecules works similarly—thus the sky scatters blue light (ν for blue is higher than red).

Atomic emission and absorption

Since the energy levels of atomic electrons are quantized, the amount of energy an atom can absorb when an electron jumps to a higher state is quantized.

An single atom can only absorb a photon of $\mathcal{E} = h\nu$ where $\mathcal{E}$ matches the energy difference between some pair of accessible states.

Likewise the energy of a photon emitted must match the energy between two states of the atom when it makes a transition to a lower energy level:

$$\Delta\mathcal{E} = h\nu.$$