Divergence 1

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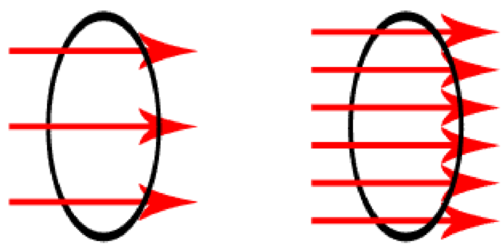
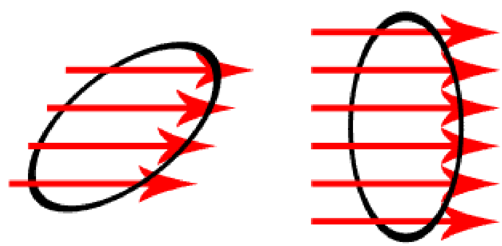
Flux

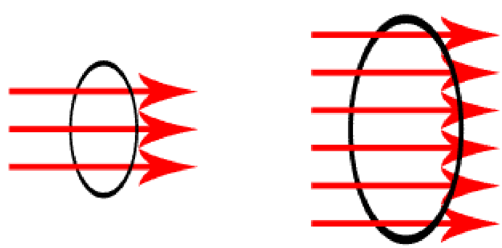
The best way to understand flux is to imagine the flow of an ideal fluid.



Flux is proportional to the density of flow.



Flux varies by how the boundary faces the direction of flow.



Flux is proportional to the area within the boundary.

See also the vector field applets at

`http://www.falstad.com/mathphysics.html`

Thanks RadRafe at wikipedia!

Flux is the volume of flow through the surface per unit time

$$\Phi_v = \int\int(\vec{\mathbf{v}} \cdot \hat{\mathbf{n}})dS = \int\int \vec{\mathbf{v}} \cdot d\vec{\mathbf{S}};$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the surface.

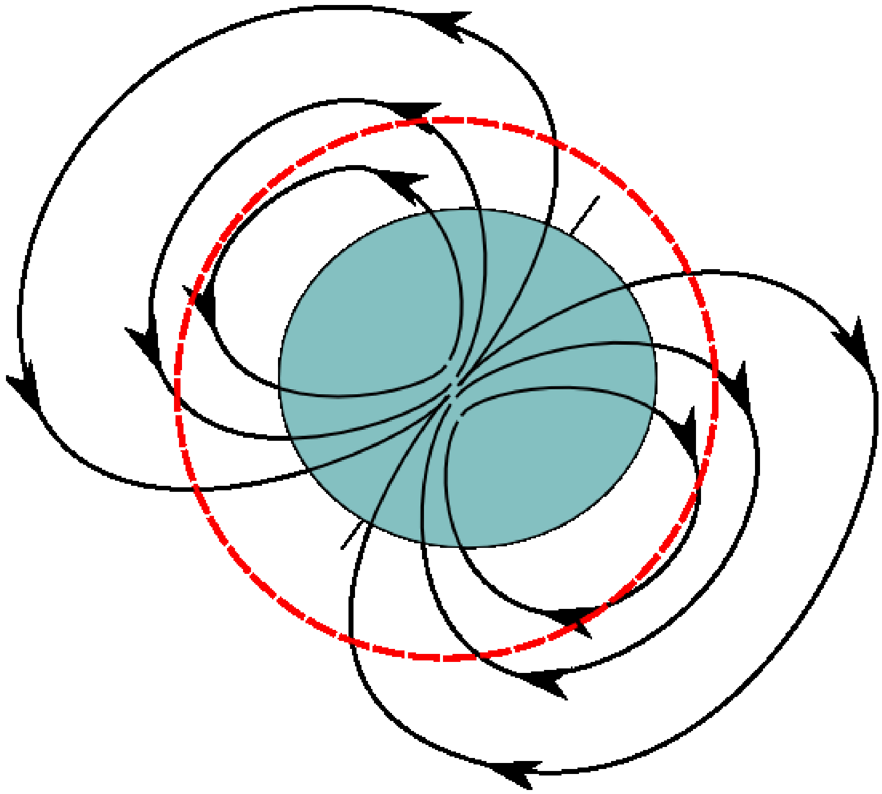
For example the field for a point charge

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{q\vec{\mathbf{r}}}{4\pi\epsilon_0 r^2]}$$

will have $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ for a spherical surface around it, and

$$d\vec{\mathbf{S}} = \hat{\mathbf{r}}R^2 d\Omega = \hat{\mathbf{r}}R^2 \sin\theta d\theta d\phi.$$

(See prob. 6.1b,c.)



The red dashed circle represents a sphere around the Earth. The total flux of the magnetic field vanishes when integrated over a closed surface surrounding a magnetic dipole.

$$\Phi_v = \int\int (\vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}) dS = 0.$$

As many field lines point in as out.

Divergence

The divergence of a vector field is the outward flux of the vector field per unit volume.

$$(\nabla \cdot \vec{\mathbf{v}}) = \frac{\mathrm{d}\Phi_v}{\mathrm{d}V}$$

Measures to what extent flow lines (field lines) *originate* or *end* within the volume.

$$(\nabla \cdot \vec{\mathbf{v}}) = 0 \text{ if no source, sink in } V.$$

For fluids, a source might be where fluid is injected into our volume of interest (faucet), a sink where fluid is withdrawn (drain).

Charges and dipoles

In electric fields, positive charge is the source of field lines, negative charge is a sink. A dipole is a source infintesimally adjacent to a sink.

Observationally, sources and sinks of magnetic fields always appear as dipoles. This means that

$$(\nabla \cdot \vec{\mathbf{B}}) = 0.$$

A magnetic monople has never been found.

Rectangular volume in cartesian coords example

Total outward flux through the sides of volume $\mathrm{d}V$ is

$$\mathrm{d}\Phi_v = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) \mathrm{d}V = (\nabla \cdot \vec{\mathbf{v}}) \mathrm{d}V$$

(p. 67)