Math Methods Homework

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This is due Monday, Apr. 9. (Original date was Apr. 6, but that is Good Friday, a holiday.) Try to get them faxed before about 3 pm if you are using a fax machine. These all can be done using the techniques outlined in chapter 17 of Snieder, except that some may have higher-order poles, see the posted notes.

Do the following using residues. For problems 2 and higher, show the contour used and the location and order of the poles. Note what parts of the contour make no contribution to the integral, and why:

1. In Snieder, section 17.3 problems f–i. See the scanned examples posted earlier for a similar example with a cosine in the numerator.

2. Find the value of
$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + 16}$$

3. Show
$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 16)^2} = \frac{\pi}{8}.$$

4. Show
$$\int_{-\infty}^{\infty} \frac{3x^2 + 2}{(x^2 + 4)(x^2 + 9)}\, dx = \frac{2\pi}{3}.$$

5. Show
$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + 4)(x^2 + 9)}\, dx = \frac{\pi}{5}\left(\frac{1}{2e^2} - \frac{1}{3e^3}\right).$$

None of these poles are particularly hard to find, but note that computer algebra systems may be useful for finding the roots of complicated denominators to locate the poles.