Three most common conventions for the Fourier transform

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In math methods, we have been looking at the Fourier transform (FT for short). There are an irritating number of conventions for this, due to possible changes in the sign of the exponential and the normalization factor which may put a factor of $\frac{1}{2\pi}$ or $\frac{1}{\sqrt{2\pi}}$ in front of the integral. Wikipedia

`http://en.wikipedia.org/wiki/Fourier_transform#Definitions`

appears to be a good reference for this. Also see Ronald N. Bracewell, *The Fourier Transform and its Applications*, McGraw-Hill. There is a third edition (2000), I have the second. See chapter 2 “Groundwork.” I have numbered the systems as he does. In all of these I have written them as time-frequency transforms.

1 Communications and signal processing convention

The *forward* transform is

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-i2\pi ft}\,dt, \tag{1}$$

where f is frequency in Hertz, t is time in seconds. The inverse is

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{i2\pi ft}\,df. \tag{2}$$

This is the most “streamlined” of the conventions, and is the one used most by Bracewell. The integration over f instead of ω eliminates the normalization constant in front of the integral.

This convention is most commonly found in engineering literature, although some engineering texts use the one below.

2 "Mathematical" convention

2.1 The standard layout

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\, dt \tag{3}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}\, d\omega. \tag{4}$$

Here ω is generally angular frequency in radians/s if dealing with an $f(t)$. This is a common convention you will find in references.

2.2 The wave propagation convention (as in Snieder)

For time and frequency this is the same as the above but *reversing the forms of the forward and inverse transforms:*

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{i\omega t}\, dt \tag{5}$$

$$f(t) = \int_{-\infty}^{\infty} F(\omega)e^{-i\omega t}\, d\omega. \tag{6}$$

For transforms in position and wavenumber, it is the "standard" forward transform as far as the exponential, but with the 2π in a different place than the mathematical convention above:

$$F(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-ikx}\, dx \tag{7}$$

$$f(x) = \int_{-\infty}^{\infty} F(k)e^{ikx}\, dk. \tag{8}$$

This is often used because if one transforms in both spatial variables ($x \to k$) and time-frequency ($t \to \omega$), then the exponential in the transform

$$e^{i(kx-\omega t)} \tag{9}$$

is the one conventionally used to represent a wave moving in the $+x$ direction. This is discussed by Snieder on p. 227 and problem 15.5d It is also a common convention in physical acoustics.[1]

Both of the above conventions do minimize the writing of factors of 2π, but at the expense of symmetry. They are most commonly found in mathematics and mathematical physics literature.

[1] Which is why I am aware of this: I did signal processing in physical acoustics.

3 Symmetric math convention

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt \tag{10}$$

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t}\, d\omega. \tag{11}$$

This is sometimes seen for symmetry's sake.

4 Doing transforms: properties and tables

The usual way to do a fourier transform is to use a table of properties and a table of standard transforms to derive your desired function without explicitly doing the improper integral.

It may be a sign of the increasing popularity of system 1 above that tables for system 2 are hard to find on the web. There is one short PDF table from the University of British Columbia at (for section 2.1 above):

`http://www.math.ubc.ca/~coombs/tch/267/ft-table.pdf`

and tables for this system are fairly easy to find in library references.

The Wikipedia reference above includes a table of transforms and properties for system 1 and system 3 above.